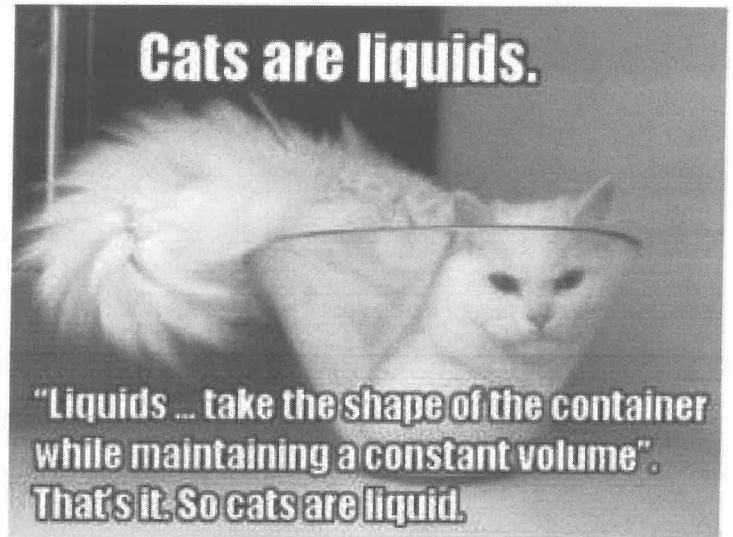


## Chapter 1: MEASUREMENT

### Conversions:

1 cm	=	10 mm
1 inch	=	2.54 cm
1 foot	=	0.3048 m
1 foot	=	12 in
1 yard	=	3 ft
1 metre	=	100 cm
1 metre	=	3.280839895 ft
1 furlong	=	660 ft
1 km	=	1000 m
1 km	=	0.62137119 mi
1 mile	=	5280 ft
1 mile	=	1.609344 km



$$A_{cylinder} = 2\pi r^2 + 2\pi rh$$

$$A_{rect.prism} = 2(lw + wh + lh)$$

$$V_{cylinder} = \pi r^2 h$$

$$V_{rect.prism} = lwh$$

$$A_{sphere} = 4\pi r^2$$

$$A_{hemisphere} = 3\pi r^2$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$V_{hemisphere} = \frac{2}{3}\pi r^3$$

$$A_{right.pyramid} = \frac{1}{2}s(\text{base.perimetre}) + (\text{base.area}) \quad A_{cone} = \pi r^2 + \pi rs$$

$$V_{pyramid} = \frac{1}{3}lwh$$

$$V_{cone} = \frac{1}{3}\pi r^2 h$$

## SECTION 1.1: IMPERIAL MEASURES OF LENGTH p. 4

In 1976, Canada adopted the **SI units** (Système International d'Unités, or Metric) to measure length. Many construction and manufacturing industries and trades continue to use imperial units, or a mix of both units. Today, Canadians typically use a mix of metric and imperial measurements in their daily lives, and so it is important to be "fluent" in both systems. We tend to know our height (ft) and weight (lb) and size of our houses (square feet) in **imperial units**. But we generally use **metric** for weather in degrees Celsius, buy gasoline and milk in litres, set speed limits in kilometres per hour (km/h), read road signs and maps displaying distances in kilometres, talk of rain and snowfall in millimetres and centimetres, and say how many pounds we can "bench" (lift in chest press). The United States and in many ways Britain continue to generally use the Imperial system (although metric is used in various industries).





In the Imperial system, more than 300 different units exist to measure various physical quantities.

Name some imperial units of measure that you know: inches  
feet      miles      yard  
teaspoon      cup      gallon

This system is NOT a decimal system. This system is based on **referents**. A **referent** is an item an individual uses as a measurement unit for **estimating**. The Imperial measurement system started in Ancient Roman Times. The current Emperor's foot length would <sup>have been</sup> the standard unit for measuring length (distances). This resulted in units that were different in different regions. In 1824, the units were standardized and became the Imperial System of Measurement. The length of a foot was standardized to equal 12 inches. Many units in the imperial system are based on the measurements of the human body.

## A. Referents for Measurement Systems

In measurement, a **referent** is a **concrete object that approximates a measurement**. Some common referents are given in the table below for the Imperial system although a referent can be almost anything that is useful.

unit	abbreviation	symbol	referent	Relationship between units
inch	in	"	Length of thumb from tip to knuckle or width of thumbnail 	
foot	ft	'	Foot length  or Length from wrist to elbow	1 ft = 12 in
yard	yd		Arm span or Width of doorway or Normal walking stride 	1 yd = 3 ft 1 yd = 36 in
mile	mi		Distance walked in 20 minutes or 2000 steps or 15-20 city blocks or Distance average person can jog in 10 minutes 	1 mi = 1760 yd 1 mi = 5280 ft

We can use referents if we don't have a measuring tool and simply want a quick estimate. If we wanted to estimate the length of a room in feet, we could walk heel to toe across the room, counting how many steps we took.

Estimate the length of your writing utensil in inches, using a referent:

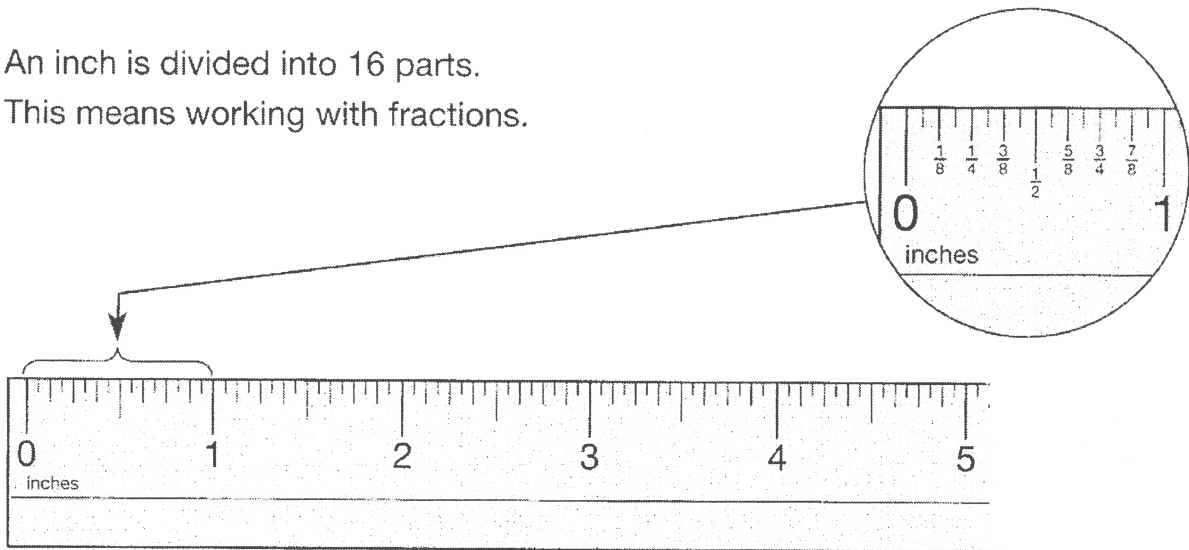
4 (We can write 4 inches; 4 in; or 4 " )

## B. Imperial Measurement

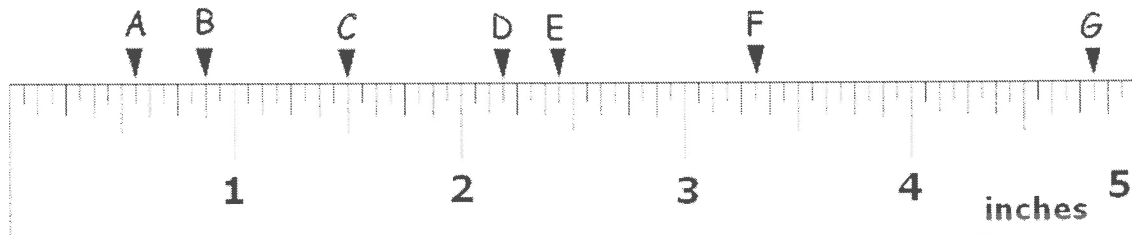
Imperial lengths can be measured with rulers, yard sticks, or calipers. Many rulers marked with imperial units show one inch divided into quarters, eighths, and sixteenths. A fraction of an imperial measure of length is usually written in fraction form, not decimal form.

An inch is divided into 16 parts.

This means working with fractions.



Notice that different lengths of lines indicate **sixteenths, eighths, quarters, and halves**.



Write the measurement in inches indicated by the following arrows, to the nearest 16<sup>th</sup> of an inch. Simplify fraction if possible.

A  $\frac{9}{16}$ "    B  $\frac{14}{16}$ "    C  $1\frac{8}{16}$ "    D  $2\frac{3}{16}$ "    E  $2\frac{7}{16}$ "    F  $3\frac{5}{16}$ "    G  $4\frac{13}{16}$ "

$= \frac{7}{8}$ "     $= 1\frac{1}{2}$ "

A common abbreviation for 5 feet 2 inches is 5 ft. 2 in. or 5' 2". (5.2 is the same as  $5\frac{2}{10}$ )

Note: 5 ft. 2 in. is NOT the same as 5.2 feet. See figure 1.

$\frac{2}{12} = \frac{1}{6} = 0.1\bar{6}$   
 $5\text{ ft } 2\text{ in} = 5.1\bar{6}\text{ feet}$

1 foot = 12 inches  
 1 foot is divided into 12 sections (inches)

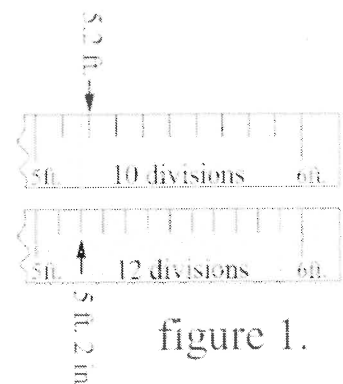


figure 1.

### C. Conversion within the Imperial System

- To convert larger units to smaller units : **MULTIPLY**

To convert a larger unit to a smaller unit (ex ft to in),

1. First check the number of smaller units needed to make 1 larger unit.
2. Next, DIVIDE that number by the number of larger units

When converting a larger unit to a smaller unit, the number increases  
(gets bigger)

Think it through logically. If 1 foot = 12 inches, to find 6 feet in inches, would we multiply or divide? Should we have a larger or smaller number of inches than feet? Try it:

$$\boxed{6 \times 12 = 72 \text{ inches}}$$

or  
 $6 \div 12 = 0.5 \text{ inches}$   
Is 6 feet = 0.5 inches? (No!)

You would expect a larger number.

Example 1: Convert 7 yd to i) ft ii) in

i) Since 1 yd = 3 ft, we must multiply 7 yd by 3 to get ft

$$7 \times 3 = 21$$

$$7 \text{ yds} = \underline{21} \text{ ft}$$

ii) On our formula sheet, we are not given the number of inches in a yard. But we know the number of inches in a foot and number of feet in a yard.

Since 1 yd = 3 ft, we must multiply 7 yd by 3 to get ft (as in i above)

Next since 1 foot = 12 inches, we must multiply 21 ft by 12 to get inches.

$$\text{or} = \frac{7 \text{ yd}}{1 \text{ yd}} \times \frac{3 \text{ ft}}{1 \text{ ft}} \times \frac{12 \text{ in}}{1 \text{ in}} = 7 \times 3 \times 12 \text{ in.} = 252 \text{ in.}$$

$$7 \text{ yds} = \underline{252} \text{ in}$$

unit analysis  
sup-8

Example 2: Convert 5' 2" (5 ft 2 in) to inches:

1. Convert 5' to inches.  $5 \times 12 = 60$  5 ft = 60 inches

2. Add 2 inches to your total in (1.↑)  $60 + 2 = 62$  5 ft 2 in = 62 inches

60 inches + 2 inches = 62 inches

- To convert smaller units to larger units : **DIVIDE**

1. First check the number of smaller units needed to make 1 larger unit.

2. Next, **DIVIDE** that number by the number of larger units

When converting a smaller unit to a larger unit, the number gets smaller  
(decreases).

Example: 12 inches = how many feet? 1

Example 3: Convert 51 in to: i) feet ii) feet and inches ii) yards, feet and inches

i) : Convert 51 in to: feet

Since 1 ft = 12 in, we must divide 51 in by 12 to get ft

51 in = 4.25 ft

ii) Convert 51 in to: feet and inches

Since 1 ft = 12 in, we must divide 51 in by 12 to get ft. 4.25 feet

- Take the whole number of feet, and multiply by 12 to get back to inches.  $4 \times 12 = 48$
- To find the remainder, subtract inches from the initial total. The remainder is the inches amount.

$$\underline{51} - \underline{48} = \underline{3}$$

- Write amount as feet and inches

51 in = 4 ft 3 in

iii) Convert 51 in to: yards, feet and inches

It may be easier to first calculate how many inches are in a yard.

$$1 \text{ foot} = \underline{12} \text{ inches. } 1 \text{ yard} = \underline{3} \text{ feet}$$

$$\text{Therefore } 1 \text{ yard} = \underline{3} \text{ feet} = \underline{3 \times 12 = 36} \text{ inches}$$

How many inches are in 1 yard?

$$1 \text{ yd} = 3 \text{ ft and } 1 \text{ ft} = 12 \text{ in.}$$

$$3 \text{ ft/yd} \times \underline{12} \text{ in./ft} = \underline{36} \text{ inches in 1 yard}$$

Since  $1 \text{ yd} = \underline{36} \text{ in}$ , we must divide 51 in by 36 to get yards.  $\underline{1.4\overline{16}}$

- Take the whole number of yards, and multiply by 12 to get back to inches.  $\underline{1 \times 12 = 12}$
- To find the remainder, subtract inches from the initial total.

$$\underline{51} - \underline{12} = \underline{39}$$

- To find the number of feet in the remainder, divide by 12.  $\underline{39 \div 12 = 3.2}$
- Take the whole number of feet, and multiply by 12 to get back to inches.  $\underline{3 \times 12 = 36}$
- To find the remainder, subtract inches from the initial total. The remainder is the inches amount.

$$\underline{39} - \underline{36} = \underline{3}$$

$$51 \text{ inches} = \underline{1} \text{ yd } \underline{3} \text{ ft } \underline{3} \text{ in}$$

The previous method is perhaps best for one-step conversions: You use one conversion factor (the equivalence between two measures or units) to convert from the one unit to the other.

But sometimes conversions are more complicated, or you're not sure which unit is "bigger". For these sorts of conversion, we use as many conversion factors as we need, setting up a long multiplication so the units we don't want cancel out. Note: this is not numbers "cancelling out", like when you're multiplying fractions. Instead, this is treating the units ("feet", "miles", "seconds", etc) as though they were numbers, and cancelling them.

How do I know which way to put the ratios? How do I know which units go on top and which go underneath? Not important. Instead, start with the given measurement, write it down complete with its units, and then put one conversion ratio after another in line, so that whichever units you don't want are eventually cancelled out. If the units cancel correctly, then the numbers will take care of themselves.

The fact that the conversion can be stated in terms of "1", and that **the conversion ratio equals "1"** no matter which value or unit is on top, is crucial to the process of cancelling units.

Setting up a unit cancellation table helps keep units straight. These tables are particularly useful when more than one unit conversion is necessary to obtain the desired unit.

### Unit Analysis

Unit analysis is a method of converting or changing measures from one unit to another by multiplying the measure by a unit conversion factor in the form of a ratio (see p. 1 conversion factors). These conversion ratios specify how one unit of measurement is related to another unit of measurement. A unit conversion factor is a fraction in which **the numerator and denominator represent the same quantity, but in different units**. The ratio can be simplified to one. There is usually a 1 in the numerator or denominator.

The fraction below is a unit conversion factor that can be used to covert miles to feet. Note that it can be simplified to one.

Conversion ratios ALWAYS equal 1.

$$\frac{12 \text{ inches}}{12 \text{ inches}} = \frac{1 \text{ foot}}{12 \text{ inches}}$$

MathBits.com

} Conversion Ratio  
(for in. and ft.)

### Unit Analysis Method:

-First, make a unit cancellation table and write the number and units you are starting with in the left top box.

-Next, find the conversion factor for the units you want. When choosing a unit conversion factor, choose the one that cancels the units you have and replaces them with the units you want.

-Write unit conversion factor as a ratio in the form of a fraction (equal to 1, where numerator and denominator represent the same quantity, but in different units. Write it in such a way that all the units will cancel, except the unit you WANT. When using unit analysis, the correct format will be such that after *mult/div*, the "have" ("from") units will cancel, and the answer will equal the "want" ("to") units.

-Then, cancel units, and multiply <sup>divide</sup>. Write the final answer with the unit you "want" (the units you are converting "to"). *top row: mult bottom row: divide*

How many inches are there in 18 feet? Use unit analysis.

18 feet	12 inches
	1 foot

$18 \times 12 = 216$ . The answer is 18 feet = 216 inches.

How many inches are in 42 miles?

42 mi	5280 ft	12 in
	1 mi	1 ft

$$42 \times 5280 \times 12 \text{ in}$$

42 miles = 2661120 inches

How many miles are in 158 400 inches?

158400 in	1 foot	1 mi
	12 in	5280 ft

$$158400 \div 12 \div 5280$$

158 400 2.5 miles

The important points are:

- Write the conversion as a fraction (that equals one)
- Multiply it out (leaving all units in the answer)
- Cancel any units that are both top and bottom

Go back and try p. 5 example 1 ii using unit conversion

Try it:

1. Convert 2 mi. to in.

$$\begin{array}{r|l|l} 2 \text{ mi} & 5280 \text{ ft} & 12 \text{ in} \\ \hline 1 \text{ mi} & 1 \text{ ft} & \end{array}$$

2 miles = 126 720 inches

$$2 \times 5280 \times 12 \text{ in}$$

2. Convert 380160 in. to mi.

$$\begin{array}{r|l|l} 380160 \text{ in} & 1 \text{ ft} & 1 \text{ mi} \\ \hline & 12 \text{ in} & 5280 \text{ ft} \end{array}$$

380 160 inches = 6 miles

$$380160 \div 12 \div 5280$$

3. Anne is framing a picture. The perimeter of the framed picture is 136 in.

a) What will be the perimeter of the framed picture in feet and inches?

$$\begin{array}{r|l} 136 \text{ in} & 1 \text{ ft} \\ \hline & 12 \text{ in} \end{array}$$

$$\frac{136}{12} = 11.3 \text{ ft}$$

$$11 \times 12 = 132$$

$$136 - 132 = 4$$

11 ft 4 in

b) The framing material is sold by the foot. It costs \$1.89/ft. The material is not sold in partial feet. What will be the cost of material before taxes?

need 12 feet  
(1.89)(12)

\$22.68

4. A school council has 6 yd of fabric that will be cut into strips 5 in. wide to make decorative banners for the school dance. How many banners can be made?

$$\begin{array}{r|l|l} 6 \text{ yd} & 3 \text{ ft} & 12 \text{ in} \\ \hline 1 \text{ yd} & 1 \text{ ft} & \end{array}$$

$$6 \times 3 \times 12 \text{ in} = 1296 \text{ in}$$

$$1296 \div 5 = 259.2$$

Can make 259 complete banners.

(not enough to make 260)

## Practice

### Hint

1 foot = 12 inches  
1 yard = 3 feet  
1 mile = 1760 yards

1. Express each measurement in inches.

a)  $6\text{ ft } 2\text{ in.} = \underline{74}\text{ in.}$   $6 \times 12 = 72$

b)  $4\text{ ft } 9\text{ in.} = \underline{57}\text{ in.}$   $4 \times 12 = 48$

c)  $2\text{ yd} = \underline{72}\text{ in.}$   $6\text{ ft } 6 \times 12$

d)  $3\text{ yd } 1\text{ ft} = \underline{120}\text{ in.}$   $9\text{ ft } 9+1=10\text{ ft } 10 \times 12$

e)  $5\text{ yd } 2\text{ ft } 3\text{ in.} = \underline{207}\text{ in.}$   $15\text{ ft } 15+2=17\text{ ft } 17 \times 12 = 204$

f)  $5\frac{1}{2}\text{ yd} = \underline{198}\text{ in.}$   $(5.5 \times 3) = 16.5' (16.5)(12)$

2. Pam is putting quarter-inch pieces of plywood in a stack. She has stacked 20 pieces so far. How high is the stack?

$20(0.25\text{ in}) = \underline{5\text{ inches.}}$

3. Express each measurement in feet.

a)  $7\text{ yd } 2\text{ ft} = \underline{23}\text{ ft}$   $21\text{ ft}$

b)  $12\text{ yd } 1\text{ ft} = \underline{37}\text{ ft}$   $36\text{ ft}$

c)  $2\text{ mi} = \underline{10560}\text{ ft}$   $(5280 \times 2)$

d)  $3\text{ mi } 5\text{ yd } 1\text{ ft} = \underline{15850}\text{ ft}$   $3(3) = 9\text{ ft}$

e)  $\frac{1}{2}\text{ mi} = \underline{2640}\text{ ft}$   $(5280) \div 2$

f)  $2\frac{3}{4}\text{ mi} = \underline{14520}\text{ ft}$   $(2.75)(5280)$

4. Ray is building a fence using panels that are sold in 8 ft lengths. The perimeter of the area for the fence measures 32 yd. How many fence panels should he buy?

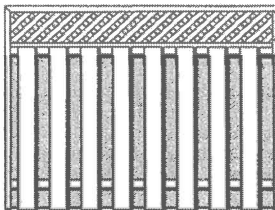
$32(3) = \underline{96\text{ ft}}$

$96 \div 8 = \underline{12}\text{ fence panels}$

5. Riley bought 50 feet of rope. He cut off pieces that total 34' 8" in length to use as tie-downs on his boat. How much rope does he have left?

$34' 8" = 34\frac{2}{3}' = 34\frac{1}{2}'$

$50 - 34\frac{1}{2} = \underline{15\frac{1}{2}\text{ feet or } 15' 8\text{ inches}}$



# SECTION: 1.3 RELATING SI AND IMPERIAL UNITS p. 16

The SI (Metric) system of measures is based on powers of ten.

Each measurement in the imperial system relates to a corresponding measurement in the SI system.

This table shows the approximate relationships between imperial units and SI units.

SI UNITS TO IMPERIAL UNITS	IMPERIAL UNITS TO SI UNITS
$1 \text{ mm} = \frac{4}{100} \text{ in}$	$1 \text{ in} = 2.54 \text{ cm}$
$1 \text{ cm} = \frac{4}{10} \text{ in}$	$1 \text{ ft} = 30.48 \text{ cm}$ $1 \text{ ft} = 0.3048 \text{ m}$
$1 \text{ m} = 39 \text{ in}$ $1 \text{ m} = 3 \frac{1}{4} \text{ ft}$ $1 \text{ m} = 3.280839895 \text{ ft}$	$1 \text{ yd} = 90 \text{ cm}$ $1 \text{ yd} = 0.9 \text{ m}$
$1 \text{ km} = \frac{6}{10} \text{ mi}$ $1 \text{ km} = 0.62137119 \text{ mi}$	$1 \text{ mi} = 1.609344 \text{ km}$

*and on front page of booklet*  
 We can use the data in the table above to convert between SI units and imperial units of measure.

To convert between units, we can use our unit cancellation table, or can mult/divide.  
 Example 1:

A bowling lane is approximately 19 m long. What is this measurement to the nearest foot?

$$19\text{ m} \approx 62\text{ feet}$$

19m	3.280839895 ft
	1m

$$19 \times 3.280839895\text{ ft}$$

(Think of a metre stick to know the length of a meter. Think of a long school ruler to think of the length of a foot (12 inches). There are about 3 feet in a metre.

The answer in feet should be more than metres.)

smaller  $\rightarrow$  larger DIVIDE  
 larger  $\rightarrow$  smaller MULTIPLY

Example 2:

After meeting in Emerson, Manitoba, Hannah drove 62 mi south and Faith drove 98 km north. Who drove farther?

(Convert one to the same unit as the other. Choice. Both in miles or both in km.)

Convert 62 mi to km: 99.7793 km is more than 98 km.

62 mi	1.609344 km
	1 mi

$$62 \times 1.609344\text{ km}$$

Hannah drove further.

Convert 98 km to mi: 60.8943 mi is less than 62 mi.

98 km	0.62137119
	1 km

$$98 \times 0.62137119$$

Hannah drove further.

### Example 3:

Alex is 6 ft 2 in tall. To list his height on his driver's license application, Alex needs to convert this measurement to centimeters.

a) What is Alex's height to the nearest centimeter?

$$\begin{aligned} 6 \text{ ft } 2 \text{ in} \\ 6 \times 12 + 2 \\ 74 \text{ in.} \end{aligned}$$

$$\begin{aligned} 74 \text{ in} \times 2.54 \\ = 187.96 \\ = 188 \text{ cm.} \end{aligned}$$

b) Use mental math and an estimation to justify that the answer is reasonable.

(Remember there are about 3 feet in a metre. And a metre is the length of a metre stick.)

$$\begin{aligned} 6 \text{ feet is about } 2 \text{ metres} \\ (6 \div 3) \\ 188 \text{ cm is } 1.88 \text{ m} \end{aligned}$$

We may need to complete more than one conversion if what we want is not on the conversion factor chart.

☞ For example, you may need to change from Imperial to SI first, then convert the SI units to the desired unit.

Remember:

x10		x10		x10		x10		x10		x10	
Kilometer	Hectometer	Decameter	Meter	Decimeter	Centimeter	Millimeter					
1000m	100m	10m	1m	1/10m	1/100m	1/1000m					
÷10		÷10		÷10		÷10		÷10		÷10	

Each metric unit gets 10 times bigger.



Each metric unit gets 10 times smaller.



Try it:

At least once a year, a truck will get stuck on the High Level Bridge in Edmonton. The bridge has a low clearance of 10' 6". A truck driver knows that her semitrailer is 3.3 m high. Will her vehicle fit under the bridge? Or will she be stopping traffic? Justify the answer.

3.3 m	3.2808 <sup>39896</sup>	12 in
	1 m	1 ft

$$= 129 \text{ in}$$

Semi trailer 129"

bridge clearance 126"

$$10 \times 12 = 120$$

$$120 + 6 = 126$$

$$(10' 6" = 126")$$

No she will not fit. She will stop traffic.

alternate method

126 in	1 ft	0.3048 m
	12 in	1 ft

$$126 \div 12 \times 0.3048 = 3.2 \text{ m}$$

bridge clearance 3.2 m  
semi trailer 3.3 m

$$126 \text{ in} = 3.2 \text{ m}$$

How could the truck driver get the trailer to fit?

Let air out of tires!

### Metric Review:

Each of the following objects have been measured with inappropriate units. Convert them to more suitable units.

1. thickness of a dime 0.00122 m <sup>convert to mm</sup>  
1.22 mm

2. height of a basketball player 2100 mm <sup>convert to m</sup>  
2.1 m

3. driving distance from Pincher Creek to Taber is 14,900,000 cm <sup>convert to km</sup>  
14.9 km.

- ① About how many centimetres are there in an inch?

1 in.  $\doteq$  2.54 cm

- ② About how many centimetres are there in a foot?

1 ft  $\doteq$  30.48 cm  $2.54 \times 12$

- ③ About how many centimetres are there in a yard?

1 yd  $\doteq$  91.44 cm  $30.48 \times 3$

Circle the greater measurement in each pair.

4

a) <sup>15.24 cm</sup> 6 in. or 10 cm

c) 10 yd or  <sup>$30 \text{ ft} \times 0.3048 = 9.144 \text{ m}$</sup>  10 m

b) <sup>1.25 m</sup> 5 ft or 125 cm  
<sub>4.1 ft</sub>

d) 25' or 8 m  
 <sub>$25(0.3048) = 7.62$</sub>

5

Circle the larger unit in each pair.

centimetre or metre

hectometre or kilometre

millimetre or decimetre

decametre or centimetre

6

How many centimetres are in 5 m?

1 m = 100 cm, so 5 m is 5 m  $\times$  100 cm/m = 500 cm

7

How many metres are in 8.5 km?

1 km = 1000 m, so 8.5 km  $\times$  1000 m/km = 8500 m

8

a) 1 cm = 10 mm

d) 45 cm = 4.5 m

b) 1 m = 100 cm

e) 1330 m = 1.3 km

c) 1 km = 1000 m

f) 45 mm = 4.5 cm